HD-THEP-96/43
HUB-EP-96/28

Relativistic QFTH-Couplings on the Worldline *

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Abstract

In the framework of the worldline path integral approach to QFTH we discuss spin and relativistic couplings, in particular Yukawa and axial couplings to spin $\frac{1}{2}$, and the case of spin 1 in the loop.

*Talk presented by M.G.Schmidt at the Dubna Joint Meeting: International Seminar on Path Integrals: Theory and Applications, and 5th International Conference on Path Integrals from meV to MeV, Dubna, Russia, 27-31 May 1996

Introduction

Interacting relativistic particles are usually described by relativistic local quantum field theory (QFTH); fields in space-time $\phi_i(x^\mu)$ are subjected to canonical quantization or they are varied in a path integral. This leads to the well-known Feynman rules. However, already in the early days of QFTH there was a different approach [1]: one considers the quantization of relativistic particles moving on a space-time worldline $x^\mu(\tau)$ and interacting with background fields. We here restrict our discussion to effective 1-loop actions, i.e. the particles are running on a closed loop.

For a massive complex scalar in the loop one has (euclidean action; singularity at $T = 0$ to be renormalized)

$$\begin{aligned}\Gamma_{\text{scalar}} &= -\log\det(-\mathcal{D}^2 + \mathbf{V}(x) + m^2) \\ &= \int_0^\infty \frac{dT}{T} e^{-m^2 T} \int_{x(T)=x(0)} [Dx(\tau)] \mathrm{Tr} P \exp\left\{ - \int_0^T \mathcal{L}_{WL}^B d\tau \right\} \end{aligned} \tag{1}$$

where the path integral describes quantum mechanics and the Schwinger T-integral takes care of relativity. The worldline Lagrangian is

$$\mathcal{L}_{WL}^B = \frac{\dot{x}_\mu^2}{4} + ig\dot{x}_\mu(\tau)\mathbf{A}_\mu(x(\tau)) + \mathbf{V}(\phi(x(\tau))) \tag{2}$$

if one has an (possibly nonabelian, matrix valued in color space) gauge background field and a "potential" (the second derivative of the QFTH-potential), e.g. $\sim \phi, \phi^2$ for ϕ^3, ϕ^4 theory resp. Indeed this "first quantization" and the background field method were used intensively in modern string theory. In particular the recurrence to relativistic particles helped to write down nonlinear σ-models for $x^\mu(\sigma, \tau)$ on the "world sheet" [2]. In the limit string tension $\frac{1}{\alpha'} \to \infty$ one should obtain the well-known local QFTH's [3]. This was studied carefully by Bern and Kosower [4] for tree and 1-loop amplitudes. The resulting rules – equivalent to but looking much different from the Feynman rules – are very effective in particular if combined with the spinor helicity formalism in QCD.

Later on Strassler [5] argued that these rules can partially be obtained from worldline path integrals. The most important ingredient in the worldline approach is the notion of worldline Green functions (GF). For the circle a convenient GF [5, 6] corresponding to the free part $\mathcal{L}_0 = \dot{x}_\mu^2/4$ is

$$G_B(\tau, \tau') = |\tau - \tau'| - \frac{(\tau - \tau')^2}{T}. \tag{3}$$

It fulfils

$$\frac{1}{2}\frac{\partial^2}{\partial\tau^2} G_B(\tau, \tau') = \delta(\tau - \tau') - \frac{1}{T} \tag{4}$$

i.e. a constant zero mode $x_\mu^{(0)}$ is taken out of $x_\mu(\tau) = x_\mu^{(0)} + y_\mu(\tau)$ and is fixed by the orthogonality $y \perp x^{(0)}$ leading to $\int_0^T d\tau y_\mu(\tau) = 0$. Thus $x^{(0)}$ is the "center of

mass"- coordinate of the loop. More general background charges $\rho(\tau)$ instead of $\frac{1}{T}$ in (4) are physically equivalent but less convenient because they break the translation invariance in τ.

Splitting $\int[Dx]$ as $\int dx^{(0)} \int[Dy]$ we can evaluate worldline path integrals contracting

$$\langle y_\mu(\tau) y_\nu(\tau')\rangle = -\delta_{\mu\nu} G_B(\tau,\tau'). \tag{5}$$

as we are used from QFTH. This allows to evaluate 1-loop effective actions very efficiently. In the Fock-Schwinger gauge

$$\mathbf{A}_\mu(x^{(0)}+y) = y^\rho \int_0^1 d\eta\eta \mathbf{F}_{\rho\mu}(x^{(0)}+\eta y)$$

and with covariant Taylor expansion of $\mathbf{F}_{\rho\mu}$ and $\mathbf{V}$ at $x^{(0)}$ one obtains, for scalar loops, [7]

$$\Gamma[\mathbf{F},\mathbf{V}] = \int dx^{(0)} \int_0^\infty \frac{dT}{T} e^{-m^2T} \sum_{n=0}^\infty \frac{(-1)^n}{n} \int [Dy] \mathrm{tr}_{\mathrm{color}} P \exp\left\{-\int_0^T d\tau \frac{\dot{y}^2}{4}\right\} \tag{6}$$
$$\times \int_0^{\tau_1=T} d\tau_2 \ldots \int_0^{\tau_{n-1}} d\tau_n \prod_{j=1}^n \left[e^{y(\tau_j)\cdot D}\mathbf{V}(x^{(0)}) + ig\dot{y}^{\mu_j}(\tau_j) y^{\rho_j}(\tau_j) \int_0^1 d\eta_j \eta_j e^{\eta_j y(\tau_j)\cdot D} \mathbf{F}_{\rho_j\mu_j}(x^{(0)})\right].$$

In the case where only the potential is present this immediately leads to the minimal number of invariants. Also for the case of gauge interactions it is the most efficient form for obtaining the heat kernel expansion. Such expansions can be used in the calculation of fluctuation corrections around quasiclassical configurations.

The effective action (euclidean notation) for gauge fields $\mathbf{A}_\mu$ coupling minimally to Dirac particles $\mathcal{L} = \bar{\psi}\mathbf{O}\psi$ with $\mathbf{O} = \mathbf{O}^+ = (\partial_\mu + ig\mathbf{A}_\mu)\gamma_\mu$ is

$$\Gamma_{\mathrm{Dirac}} = \log\det \mathbf{O} = \frac{1}{2}\mathrm{Tr}\log \mathbf{O}\mathbf{O}^+ = \frac{1}{2}\mathrm{Tr}\log(-D_\mu^2 \mathbf{1} + g\sigma_{\mu\nu}\mathbf{F}_{\mu\nu}). \tag{7}$$

This contains a Dirac matrix valued potential but otherwise is similar to the scalar theory ("second order formalism"). One can substitute the Dirac matrix trace by a Grassmann variable integration over $\psi(\tau)$ [6, 5, 9]

$$\Gamma_{\mathrm{Dirac}} = -2\int_0^\infty \frac{dT}{T} \int_{x(0)=x(T)} [Dx] \int_{\psi(0)=-\psi(T)} [D\psi] \mathrm{tr} P \exp\left\{-\int_0^1 L_{WL}^D d\tau\right\} \tag{8}$$

where $\mathcal{L}_{WL}^D = \mathcal{L}_{WL}^B + \mathcal{L}_{WL}^F$ now contains besides the bosonic piece (2) a fermionic part

$$\mathcal{L}_{WL}^F = \frac{1}{2}\psi_\mu \dot{\psi}_\mu(\tau) - ig\psi_\mu(\tau)\psi_\nu(\tau)\mathbf{F}_{\mu\nu}(x(\tau)) \tag{9}$$

with a corresponding Green function

$$\langle \psi_\mu(\tau)\psi_\nu(\tau')\rangle = \frac{1}{2}\delta_{\mu\nu} G_F(\tau,\tau') = \frac{1}{2}\delta_{\mu\nu}\mathrm{sign}(\tau-\tau'). \tag{10}$$

Surprisingly $\mathcal{L}^D_{WL}$ is supersymmetric (a remnant of local SUSY), i.e. invariant under $\delta x_\mu = -2\eta\psi_\mu, \delta\psi_\mu = \eta\dot{x}_\mu$ with a constant Grassmann parameter η, though the boundary conditions break the SUSY. Introducing superfields in superspace $\hat{\tau}(\tau,\theta)(\int d\theta\theta = 1) : X_\mu(\hat{\tau}) = x_\mu(\tau) + \sqrt{2}\theta\psi_\mu(\tau)$ and a superderivative $D = \frac{\partial}{\partial\theta} - \theta\frac{\partial}{\partial\tau}$ the worldline action can be written as

$$\int_0^T d\hat{\tau}\mathcal{L}^D_{WL} = \int_0^T d\tau \int d\theta \hat{\mathcal{L}}^D_{WL} = \int d\hat{\tau}\left(\frac{1}{4}DX_\mu D^2 X_\mu - igDX_\mu \mathbf{A}_\mu(X)\right). \tag{11}$$

Thus the formalism is manifestly supersymmetric [6, 8, 10] and one can get the effective action with Dirac particles in the loop from the one with scalars by just substituting super Green functions

$$\hat{G}(\hat{\tau},\hat{\tau}') = G_B(\tau,\tau') + \theta\theta' G_F(\tau,\tau') \tag{12}$$

in the bosonic calculation. This also explains the so-called "chain"-rule: substitute (index-) closed chains of $\dot{G}_B(\tau,\tau') = \frac{\partial}{\partial\tau}G_B(\tau,\tau') = \mathrm{sign}(\tau-\tau') - 2(\tau-\tau')/T$ by the same chain minus the corresponding chain of G_F's in the Dirac case. Note that SUSY unbroken by the b.c.'s would imply $\dot{G}_B = G_F$ and hence a vanishing of such terms.

Further spin $\frac{1}{2}$ couplings

Curiously the Yukawa coupling $\mathcal{L}^{\mathrm{Yuk}} = \bar{\psi}(-i\lambda\phi)\psi$ was not translated to the worldline formulation until our recent work with M. Mondragón and L. Nellen [11]. In a dimensional reduction approach it can be interpreted as a gauge coupling in a fifth dimension (after some redefinition of γ_μ). Introducing a new superfield $X_5 = \tilde{x}_5 + \sqrt{2}\theta\psi_5$ or its more convenient superderivative ($x_5 \equiv -\dot{\tilde{x}}_5$)

$$\bar{X} = \sqrt{2}DX_5 = \sqrt{2}\psi_5 + \theta x_5 \tag{13}$$

the (super) worldline Lagrangian is extended by

$$\begin{aligned}\int d\theta \hat{\mathcal{L}}^{\mathrm{Yuk}}_{WL} &= \int d\theta\left(\frac{1}{4}\bar{X}D\bar{X} + i\lambda\phi(X_\mu)\bar{X}(\hat{\tau})\right)\\ &= \frac{x_5^2}{4} + \frac{1}{2}\psi_5\dot{\psi}_5 + i\lambda\left(x_5\phi(x) - 2\psi_5\psi_\mu\partial_\mu\phi(x)\right).\end{aligned} \tag{14}$$

Integrating out the auxiliary field x_5 we obtain

$$\frac{1}{2}\psi_5\dot{\psi}_5 + \lambda^2\phi^2 - 2i\lambda\psi_5\psi_\mu\partial_\mu\phi(x). \tag{15}$$

Substituting ϕ by m/λ gives the well-known mass term [12].

For a more systematic treatment [13] consider now in analogy to eq. (7) a Dirac operator (euclidean)

$$\mathbf{O} = (\partial_\mu + igV_\mu + ig\gamma_5 A_\mu)\gamma_\mu - im - i\lambda\phi + \gamma_5\lambda'\phi' \tag{16}$$

with scalar (ϕ), pseudoscalar (ϕ'), vector (V_μ) and axial vector (A_μ) couplings. The operator $\mathbf{O}^+$ has a change in sign in the A_μ and m, ϕ terms. Thus we substitute (7)

by

$$\Gamma = \mathrm{Tr}\log\mathbf{O} = \frac{1}{2}\mathrm{Tr}(\log\mathbf{OO}^+) + \frac{1}{2}\mathrm{Tr}(\log\mathbf{O} - \log\mathbf{O}^+) \tag{17}$$

with

$$\begin{aligned}\mathbf{OO}^+ \quad = \quad & -D^2 + g\sigma_{\mu\nu}V_{\mu\nu} + g_5\gamma_5\sigma_{\mu\nu}A_{\mu\nu} + i\lambda\gamma_\mu\partial_\mu\phi \\ & -\lambda'\gamma_5\gamma_\mu\partial_\mu\phi' + 2(im + i\lambda\phi - \gamma_5\lambda'\phi')ig_5\gamma_5\gamma_\mu A_\mu + m^2 \\ & +2m\lambda\phi + \lambda^2\phi^2 + \lambda'^2\phi'^2\end{aligned} \tag{18}$$

where $V_{\mu\nu}, A_{\mu\nu}$ are the field strengths. The derivative of the second term in (17) with respect to the background field $U(U = A, \phi'$ in the following) can be written as

$$\frac{\delta}{\delta U}\mathrm{Tr}(\log\mathbf{O} - \log\mathbf{O}+) = \mathrm{Tr}\left(\left\{\frac{\delta\mathbf{O}}{\delta U}\mathbf{O}^+ - \mathbf{O}\frac{\delta\mathbf{O}^+}{\delta U}\right\}\frac{1}{\mathbf{OO}^+}\right). \tag{19}$$

This "second order" formalism can be translated to the worldline formulation with a purely Grassmann even Lagrangian if one introduces two additional superfields $\bar{X}$ and X' appearing in the coupling to scalars / pseudoscalars as in (14). The new piece of the worldline action is particularly simple if expressed in superfields. We proposed [13]

$$\begin{aligned}S_{WL} \quad = \quad & \int d\tau d\theta\Big[\frac{1}{4}DX_\mu D^2X_\mu + \frac{1}{4}\bar{X}D\bar{X} + \frac{1}{4}X'DX' \\ & +i\lambda\phi(X)\bar{X} + i\lambda'\phi'(X)X' - igDX_\mu V_\mu(X) + g_5\bar{X}X'DX_\mu A_\mu(X)\Big]\end{aligned} \tag{20}$$

with the original worldline supereinbein field $\Lambda = e + \sqrt{e}\theta\chi$ gauge fixed to $e = 2,\ \chi = 0$ on the circle. The form of the axial coupling with both $\bar{X}, X'$ involved is surprising. Written out in components this is a lengthy expression [13].

The imaginary part (19) can be worked out and also translated to worldline language

$$\Gamma'_U = \frac{\delta}{\delta U}i\mathrm{Im}\Gamma = -2\int_0^\infty \frac{dT}{T}\int [DX_\mu][D\bar{X}][DX'](-1)^F\Omega_U e^{-S_{WL}} \tag{21}$$

with

$$\Omega_{\phi'} = -i\lambda'\sqrt{2}\int d\tau d\theta\theta(-\frac{1}{4}D^2X_\mu DX_\mu + \frac{1}{4}\bar{X}D\bar{X})X' \tag{22}$$

and a similar expression (integrated product of two superoperators appearing in S_{WL}!) for Ω_{A_μ}. (21) besides the action term contains the Witten index operator $(-1)^F$ changing the fermionic b.c.'s from antiperiodic to periodic, and the Ω_U. The Γ'_U have to be integrated over the field U for getting the action (An alternative way was given in [14].). Interestingly $(-1)^F$ plays the role of γ_5, a well-known fact in the discussion of anomalies [15]. The change to periodic b.c.'s allows for constant Grassmann modes ψ_0. Grassmann integration then requires each of the modes to be present once. This leads to ϵ-tensors.

We have tested our worldline action by calculating [13] a collection of amplitudes first in the conventional Dirac-Feynman, then in second order, and finally in the worldline formalism. In particular, we could reproduce PCAC and the axial anomaly; we can also derive higher order terms in the 1-loop effective action.

In recent publications [14, 16] it was argued that $\mathbf{O}$ and $\mathbf{O}^+$ can be arranged in a 8×8 spinor matrix corresponding to a 6-dimensional space, thus explaining the need for our fields $\bar{X}, X'$. The 6-dimensional γ-matrices can be substituted by Grassmann ψ's and our worldline Lagrangian is reproduced and generalized in an elegant way. There is a subtle technical point [13, 17] about either integrating out the auxiliary fields x_5, x_6 belonging to $\bar{X}, X'$ or contracting them in Green functions if the correct field theory results are to be obtained.

Spin 1 in the loop

The case of massless spin 1 gauge bosons coupling to a gauge boson background is particularly important in QCD applications. In the covariant 't Hooft-Feynman background gauge the 1-loop action has the Schwinger form

$$\Gamma(A) = \frac{1}{2} \int_0^\infty \frac{dT}{T} \mathrm{Tr} \exp(-\hat{h}T) \tag{23}$$

with the (color and Lorentz) matrix valued Hamiltonian

$$\hat{h}^{ab}_{\mu\nu} = -D^{ac}_\rho D^{cb}_\rho \delta_{\mu\nu} - 2ig\mathbf{F}^{ab}_{\mu\nu}(x). \tag{24}$$

One can evaluate this by a "bosonized" worldline path integral including a matrix valued potential but motivated by the spin $\frac{1}{2}$ case one might try to substitute Lorentz (and eventually also color) indices by Grassmann variables. Building on an earlier proposal [5] M. Reuter and the present authors have shown [18] that a worldline Hamiltonian

$$\hat{H}_0 = (\hat{p}_\mu + gA_\mu(\hat{x}))^2 - :\hat{\bar{\psi}}_\mu (2igF^{ab}_{\mu\nu})\hat{\psi}_\nu : \tag{25}$$

has the required properties. Here $\hat{p}_\mu = -i\frac{\partial}{\partial x_\mu}$, $\hat{\bar{\psi}} = \frac{\partial}{\partial \psi_\mu}$, and $\bar{H}_0$ acts on forms $\phi(x, \psi)$ depending on x and a set of classical Grassmann variables ψ_μ. The second term in (25) needs to be anti-Wick ordered. $\hat{H}_0$ then acts on 1-forms (in ψ) just as the operator (24). Thus one has to project onto the one-forms. This requires the introduction of a kind of mass term $C\hat{\psi}_\mu\hat{\bar{\psi}}_\mu$ in H_0 with C to be taken to infinity at the end. Furthermore, a GSO type projector $(1-(-1)^F)$ has to be applied. Besides this the anti-Wick ordering has to be expressed in terms of a Weyl ordered $\hat{H}_{\mathrm{Weyl}}$ appropriate for transcription to a worldline path integral with midpoint rule. Altogether we obtain an action

$$\Gamma(A) = -\frac{1}{2} \lim_{C\to\infty} \int_0^\infty \frac{dT}{T} e^{-CT(D/2-1)} \int [Dx_\mu] \frac{1}{2} \left(\int_{\mathrm{antiper.}} - \int_{\mathrm{per.}} \right) [D\psi_\mu][D\bar{\psi}_\mu]$$
$$\times \mathrm{Tr} P \exp\left[-\int_0^T d\tau \left\{ \frac{\dot{x}_\mu^2}{4} + ig\dot{x}_\mu A_\mu + \bar{\psi}_\mu \left((\partial_\tau - C)\delta_{\mu\nu} - 2ig\mathbf{F}_{\mu\nu} \right) \psi_\nu \right\} \right] \tag{26}$$

and fermionic Green functions

$$G^C_{\substack{\text{per.}\\ \text{antiper.}}} = -\left[\theta(-\tau) \pm \theta(\tau)e^{-CT}\right] e^{C\tau}/(1 \mp e^{-CT}) \tag{27}$$

similar but not identical to the expressions in [5]. We should stress that (26) is the result of a rigorous derivation, will lead to correct gluon amplitudes, and thus is a good starting point for deriving Bern-Kosower type 1-loop rules. As a simple application we have rederived [18] the 1-loop action in a constant pseudo-abelian background.

Considering the multiloop case it is not so obvious that introducing Grassmann variables to represent spin is an equally efficient way as it is for spin $\frac{1}{2}$. A 2-loop derivation directly from string theory would be very illuminating. For first steps into this direction see [19]. The multiloop formulation of QFTH in the worldline language will be subject to another contribution in these proceedings.

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